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STATISTICS OF SCALING SYSTEMS

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Abstract:

Traditional linear models of combat treat attrition as a continuous function of time. However, recent thinking on the nature of warfare has it that combat is an inherently non-linear situation. Non-linearity in many fields has been shown to generate discontinuous functions to describe various quantities as functions of time, which often show scaling similarities. Such functions are often well-described by fractal models. Here, the possibility of using a fractal to describe attrition is examined. Using such a formalism, an argument can be constructed from historical data that the front over which combat occurs evolves toward an attractor with a fractal dimension of 1.685. In this way, much as for many chaotic systems, the specific details of the combat evolution may not be important to the end result, given reasonable tactical behaviour. Using a model of combat between individual troops, it is shown that combat casualties tend to be intermittent, that is, come in bursts, and that as a result the statistical moments of the data scale. This means that, for example, the variance of the casualty ratio increases as the size of the units involved in the combat decreases. This has significant implications for the statistical understanding of combat, particularly that combat statistics "scale," providing justification for modelling combat in terms of aggregated units. Given the high degree of variability of small-scale combat, it seems that modelling such combat explicitly for analysis purposes (rather than for command training) may be of limited value unless large numbers of simulations are run. Also, historical data suggests that reasonably fat-tailed distributions (hence with greater extremities than normal distributions) must be produced by a model if it is to accurately represent reality. Significantly, fractals may be used to produce fat-tailed probability distributions. Should real combat statistics possess fat-tailed probability distributions, characterisable by fractal models, this has significant implications for estimating the risk associated with warfighting.

Defence Operational Support Technology Establishment
Auckland, New Zealand

EXECUTIVE SUMMARY

Background

As part of the Tekapo Manoevres programme, DOTSE has been commissioned to provide support for wargames and simulations designed to explore the issues which arise with the restructuring of the Army and the purchase of new equipment. In order to do this, DOTSE has been investigating several combat simulation tools which provide methods for determining the outcome of combat on different scales of representation. During the course of this, the author suggested that some thought be given to how the statistics of such models should vary with the scale at which the combat is being examined. This is the resulting work.

Sponsor

New Zealand Army.

Aim

To show how the non-linearity inherent in the dynamics of combat can affect the statistics of the outcome.

Results

It is shown how a fractal model can be used to describe attrition. It is further shown that certain fractal-like properties of combat statistics suggest that the variance of the statistics (second order moment) for combat should increase as the units involved become smaller for some range of unit sizes, provided that the units of each size are of comparable composition (e.g. all similarly armed infantry, for example).

Contents

1. Introduction

Combat simulations are often used by defence forces both for analysis and training. Thanks to advances in computer technology, combat situations may be relatively easily modelled in pain-staking detail on a PC computer. However, combat must generally be considered to be a complex system which evolves in an unpredictable way (even if the end results themselves tend to be predictable). Experience in a wide range of fields for which complex systems exist (e.g. geophysics) has shown that increasing the detail of the model does not necessarily improve its ability to reproduce reality, but often simply reflects the emphasis placed on various aspects by the designer. Additionally, many detailed models require tweaking or fudge factors in order to reproduce the observed behaviour.

This complexity makes modelling of a combat situation difficult, particularly because the model designer must assume knowledge of how the participants will behave in a given situation. Thus there is a real danger that the model will simply reproduce the pre-conceived ideas of the modeller.

The degree of detail modelled raises further issues. For example, detailed terrain data is often used in such simulations. Obviously the use of these maps assumes that the details of the terrain and the participants' movements over that terrain are crucial to the simulation. However, the number of ways the forces may be arranged and subsequently traverse the map is large and the terrain for a given battle will in reality be almost arbitrary, since (for simulations of small-scale encounters anyway) the combat could occur at any number of locations. But if this degree of detail is vital to the outcome of the simulation, then the analyst's job becomes almost impossible, since the performance of a particular force must be determined over an almost infinite number of variations of terrain.

On the other hand, if statistically similar results always occur for reasonably similar terrain, then it may be possible to reduce the problem to one of generalised behaviour. Such a step requires an assumption that combat for units of a particular size always evolves in the same sort of way, regardless of the variations within broad terrain classes. If so, then the combat may either be modelled with simplified automaton models, whose behaviour is determined by some set of rules rather than by explicit instruction by the modeller, or more simply still the combat may be modelled in a purely statistical way.

Interestingly, the fact that military conflicts throughout history show consistent patterns has been noted by some analysts (e.g. Dupuy 1987), suggesting that perhaps there exists some basic law of combat behaviour. Intuitively combat would seem to be a chaotic activity, if for no other reason than that in the course of a particular battle the chain of command is likely to become dysfunctional as the officers and NCOs who make it up become incapacitated or killed. Mathematically chaotic systems often possess chaotic attractors to which the dynamics evolve, so that it is worth considering that if combat is a chaotic dynamical system, then perhaps the statistics of combat are drawn towards such an attractor. Usually chaotic attractors require fractal geometry to describe (see Tsonis and Elsner, 1989), and furthermore the statistics of the system itself may display fractal traits, such as power-law spectra, scaling of the statistical moments, and fat-tailed probability distributions.

Here, the possibility that combat obeys fractal statistics is explored, both to demonstrate the consequences of this, and to consider evidence for the existence of such behaviour using both historical data and data from a high-resolution combat model. Two things are considered: What are the consequences of using a discontinuous function such as a fractal to describe the attrition rate, and is there any reason to suspect that such a discontinuous function is more appropriate for attrition models than a continuous one?

Note that fractal statistics have successfully been applied to other complex systems, for example, Mandelbrot (1997, 1999) has recently shown that they apply for the sharemarket, while meteorological systems have been known to produce this kind of statistics since the work of Kolmogorov (1941) first predicted power-law spectra for turbulence. Additionally, cellular automaton models, which have become increasingly widely used for modelling the complexity of warfare (Hunt, 1998; Hoffman and Horne, 1998; Ilachinski, 1997; Woodcock *et al*, 1988), may also demonstrate power-law spectra (Bak P., Chen K., and Creutz M., 1989; Bak P., Chen K., and Tang C., 1990), so that if one is to accept automaton models for combat, there must also be an implicit acceptance that combat statistics are fractal. But unlike automaton models (the results of which are sometimes difficult to relate to a useful real-world context (fractal statistics may be related to tangible probability distributions, which may be incorporated into coarse-grained models.

Finally, it is noted that fractal statistics were applied in areas such as finance for a reason: they more realistically characterise risk. Specifically, **the properties of discontinuity and intermittency significantly affect risk.**

2. Fractal indices for combat

2.1 Is a combat front a fractal?

As a starting point, it may be useful to try to construct a picture of what happens during a battle to cause the statistics to become fractal. To support the argument that at least some combat situations evolve towards an attractive state regardless of the initial conditions, use is made of an observation derived from historical analysis of infantry battles by the UK's Centre for Defence Analysis (CDA). This has been incorporated by CDA into a simple model of infantry close combat (SMICC) (Hall, Wright and Young, 1997), which is a purely statistical model. One result obtained by CDA from the historical analysis was that a "force ratio factor" could be defined as:

$$F = (\text{Number of attackers/Number of defenders})^{0.685} \qquad (1.1)$$

where F is used as a multiplier for the base number of casualties of the attacking force derived from other input variables.

The force factor's physical interpretation is that an outnumbered force will have a larger target area to shoot at than its opponents, and so finds it easier to acquire targets.

Consider this problem geometrically by relating expression 1.1 to a combat frontage, *S*, with a length equal to the dimensionless quantity *F* multiplied by a scalar constant $L_{defenders}$ with units of length. Here, $L_{defenders}$ may be loosely interpreted as the initial length of the front of the defending force.

Matching shooters to targets for equal numbers of attackers and defenders along the front gives a one-to-one relationship, and $S = L_{defenders}$.

However, for an unmatched number of attackers and defenders, the non-linear nature of the force factor suggests that the matching of shooters to targets is more complicated. Consider figure 1A, showing two forces facing each other and stretched between two points above or below which neither can occupy. For equal-sized forces, the front ranks of each match up, and this remains so even if part of one line is pushed back.

However, in the figure the red line is fatter, representing that it has deeper ranks of troops. If the blue line becomes buckled under the weight of red's superior numbers, as in figure 1B, the red force can stretch around the bend by reducing the depth of its ranks, thus matching more of its troops to the blue force's front rank.

If red's superior force continues to buckle the blue line, then eventually its troops will have stretched around the blue force to the point where the depth of the ranks of each force are the same. At this point, red is making full use of its numerical advantage, and, presents its largest target area.

As the ratio of attackers to defenders, *R*, gets bigger, the pattern into which the combat front may evolve becomes more complex, since ever more kinks in the blue line are required to stretch the red force to its full extent. It may be imagined, then, that there exists some complicated geometrical object that the curve describing the combat front tends to as the ratio *R* becomes larger. Thus the actual combat front may be viewed as a coarse-grained approximation of this geometrical object, where *R* determines the coarseness. What is meant by this is that if *R* were used to define a measuring stick, say, a = $1/R$, to measure the length of this object, then *N* sticks of length a would be required to cover the object, without overlapping. The length of the object using such a coarse-grained measure is then $S = N$ (a, which is the length of the real-world combat front.

Since the geometrical object discussed above reveals increasing structure as the measure a decreases, it is a fractal. Though the definition of a fractal appears to vary between different groups of workers (Saucier 1991), the requirement for structure on all scales appears common to all.

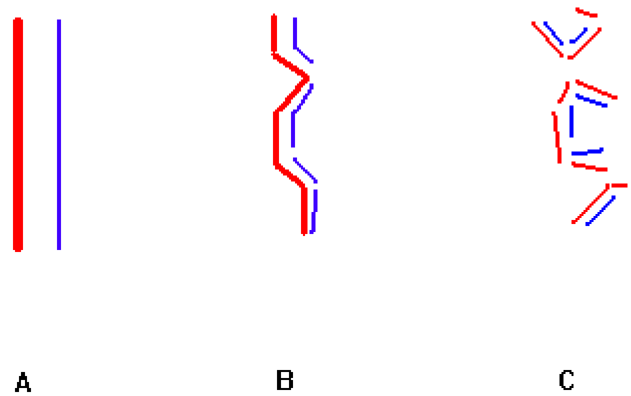

Figure 1: Idealised combat evolution.

The fractal itself may be imagined to be a combat front which meanders to such an extent that it would take an infinite amount of time to traverse. Mandelbrot (1983) defines the fractal dimension of such a curve to be:

$$D = \lim_{a \to 0} \frac{\log(N)}{\log\left(\frac{1}{a}\right)} \tag{1.2}$$

where the force factor defined in equation 1.1 suggests D should have a value of 1.685. The existence of this force factor supports such a phenomenological model, where the combat always evolves towards the same situation, regardless of the details of the evolutionary paths that the combat takes. The fractal described above would thus be an attractor for the dynamical system of the combat.

2.2 Scaling and form of casualty statistics

The nature of combat statistics provides a more tangible question than heuristic arguments about how combat evolves. Models of combat between forces of arbitrary size seek to provide either a casualty figure or casualty rate for the engagement as one measure of the outcome. Often, particularly for wargames, the forces are treated as aggregated units, so that the detailed outcomes of the individual combat between sub-units of each force are assumed to be describable by a single function. For large-scale forces, the attrition rate is often assumed to be continuous (e.g. Lanchester) within a single battle. This assumption is likely to be reasonable for large enough forces, but as the scale of the forces becomes smaller, casualties must be determined in another way. This may be done by explicitly describing the combat outcome for each element of the force. However, this is much more cumbersome than simply producing a casualty figure for the entire unit. If the dependence of such a casualty figure on the size of the force and its variance could be understood, then a model for aggregated units of any size could be constructed.

This section considers the use of multiscaling (or multifractal) statistics (see Davis *et al*., 1994, for a good overview) to describe the scaling of combat.

Consider a real, non-negative scalar field R of discrete points R_i which describe the distribution of the casualty rate in time. If such a field is multiscaling, then its behaviour can be described by:

$$\left\langle R_i^{\,q} \right\rangle \propto \left(\frac{t}{T} \right)^{K(q)} \qquad 1.3$$

where the angled brackets represent an ensemble average, t is the temporal resolution at which the distribution is being examined, T is the "outer" scale of the scaling range, and $K(q)$ is a non-linear function of the order of the statistical moments, q. Generally T is the point where the scaling breaks down.

If combat statistics are indeed multiscaling, there are several properties that must be incorporated into combat models.

Firstly, multiscaling fields are typically non-Gaussian, exhibiting fat-tailed distributions, meaning that they contain more extreme values than a normal distribution. This is an important property not only because trainee commanders must learn to deal with unexpected catastrophes, but also because analysts must plan for such events. Mandelbrot (1999) makes this point succinctly, by pointing out that it is all very well to have a model with conventional statistics which describes sharemarket behaviour 95 per cent of the time, but if that other 5 per cent is when the market crashes, then the model's usefulness is somewhat dubious. By using fractal statistics for combat, we are endeavouring to incorporate the military equivalent of a sharemarket crash.

Interestingly, the CDA workers who developed SMICC have fitted a lognormal distribution to the historical data on which their model is based. However, the author knows of no theoretical justification for using a lognormal distribution, so it should be seen as an approximation. Furthermore, a lognormal curve has a fatter exceedence tail than a normal distribution, suggesting that the actual distribution of combat data to which the lognormal approximation was made had a fat-tailed distribution of some kind.

Secondly, multiscaling fields tend to be intermittent. In terms of combat data, this means that most of the casualties occur around specific points in time, rather than continuously. Thus the casualty rate must be expected to be far from constant.

Finally, if the casualty rate distribution exhibits temporal scaling, this scaling can be used to determine the distribution of casualties for units of a given size. This is because small units run out of troops sooner than larger units.

If the unit fights only until it reaches a certain casualty ratio, then the typical duration of the combat, t, will be longer the larger the unit is. If a mean combat duration time $\tau(L)$ exists for units of size L, then $\tau(L)$ can conveniently be taken to be a fixed time step for the model.

Thus for a particular unit of size L, the casualties incurred after a period τ will be τR_τ, where R_τ is the average casualty rate during the period τ and is a random variable from a probability distribution with a variance proportional to $(\tau / T)^{K(2)}$. This implies that the variance of the casualties depends on the size of the unit.

The scaling of the variance is such that the variance is smaller in relative terms as the unit size gets larger. Thus this property might be incorporated into a simple statistical model of combat.

As an example, CDA's SMICC considers a number of factors (including the force ratio factor discussed in the preceding section) to determine a mean casualty figure, which with a given fixed variance assumes a lognormal distribution for the casualty ratio. However, the possibility of the variance being a scale-dependent parameter is not taken into account.

A model of this type might be simply modified to take into account scaling behaviour by making the variance dependent on the size of the force involved. Figure 2 demonstrates how this may be done.

The value of $K(2)$, which determines the scaling of the variance, must be experimentally determined and unfortunately the available data is not sufficient to do this. For the example in figure 2, $K(2)$ is simply guessed as 0.5. Note that the value of $K(2)$ may also vary depending on general classes of conditions in which the combat is being fought, for example, various terrain classes or types of weapons employed. The figure demonstrates how the scaling of the variance affects the exceedence distributions for such a model for various sized units. Note that this figure is not meant to literally show the distribution for a squad, section, etc (this must be determined empirically), but to show how the distribution may vary for a hierarchy of units whose size is related to each other by roughly these proportions.

From the figure, the outcome is less variable for a large force than for a small force. This is related to the fact that a mutliscaling casualty distribution implies casualties come in bursts. Clearly, a severe burst of casualties can potentially eliminate a much larger portion of a small force than a large one, if for no other reason than a larger force is spread over a bigger area, and it is therefore more difficult to engage a large portion of it.

Conversely, a small unit has the possibility of destroying a large portion of an opposing small unit with a single volley, and so might avoid suffering a high level of casualties itself.

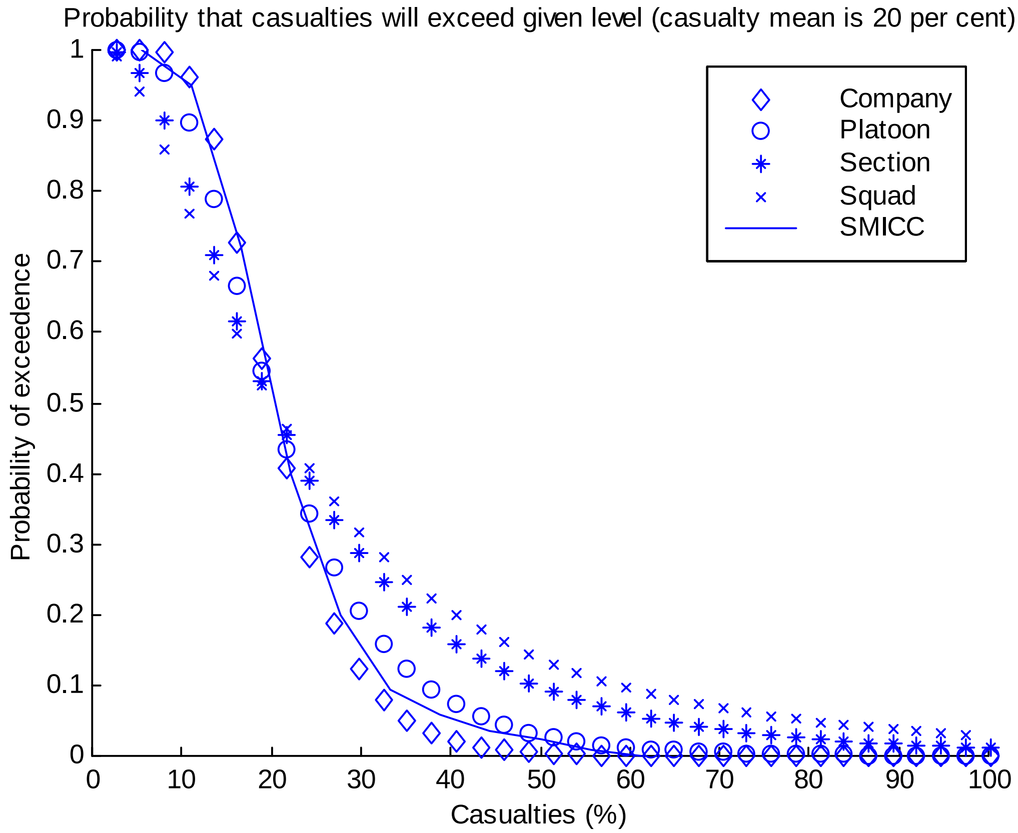

Figure 2: A possible distribution of casualties as a function of unit size, compared with the single distribution SMICC uses.

3 Evidence for the burst-like nature of combat

3.1 Analysis of model data and simulations

Though the use of concepts like casualty bursts and higher variability of small-scale combat may be qualitatively justifiable, the use of a scaling scheme for combat statistics as proposed in the previous section requires empirical justification. Unfortunately, suitable data is not only difficult to obtain, but its quality is usually poor by scientific standards. Additionally, the fractal indices which describe the statistics may vary depending on the circumstances of the combat (e.g. terrain or weapon types).

In this section, a brief examination is made of results obtained from a combat simulation model in the context of the suggestions made in the previous sections. The model, called Integrated Unit Simulation System (IUSS), was developed for the US Army Natick Research, Development and Engineering Centre by Simulation Technologies of Dayton, Ohio.

The model represents the combat down to the level of individual soldiers. As discussed above, such combat models may become dubious the longer the combat has to evolve, because assumptions must be made by the modeller about the evolution. Here, an effort was made to keep the complexity of the combat to a minimum to avoid this difficulty. By having a short-duration and small-scale combat, the need for more explicit modelling of the evolution of the combat was partially eliminated.

The combat began with a blue force attacking in roughly a line formation against a red force in a defensive line. The forces exchange fire, then part of the blue force advances on the red position, with the rest providing cover. These events were designed to mimic a generic section-level infantry attack. For the example here, the strength of the red force was nine soldiers, and the blue force 18. This served as a simple model for combat evolution, with the attacker advancing to an appropriate engagement range, fighting at that range, then advancing to a new engagement range, and so on. The simulations showed that (not surprisingly) most of the casualties occurred when blue first appeared, and then as the advance began.

Casualties were recorded with a 10 second sampling rate (i.e. the number of casualties every 10s), and the combat was deemed to be over once one side had lost more than 50 per cent of its original strength at the end of any 10s step.

Though the simulation behaved like a fight between pre-programmed robots, it was sufficient to illustrate the intermittent nature of the combat, as figure 3 shows.

Note particularly that: a) the casualty rate fluctuated strongly. b) casualties tended to be clustered together in time. However, figure 4 shows the distribution of casualty outcomes using the IUSS simulations was much closer to a normal than the lognormal distribution suggested by the historical data used for the SMICC model. This is likely to be because of the lack of human factors such as morale. Since each run produced the same behaviour each time, the only variability was due to the randomness in determining kills. Neither side, for example, ran away because its leader was killed, and such vagaries of human nature are likely to lead to extremities in the statistics.

The data obtained from this model was used to illustrate the concept of scaling. This was done by imagining multiple such encounters as shown in figure 3 occurring in succession, simulating further casualties as reinforcements arrive for both sides. The simulation described above was run several times and the data sets “joined” to make a single set. Figure 5 shows the resultant time series as though part of a continuous series of events. Here, the data is displayed in terms of casualties per unit time. The data is then aggregated twice by successively reducing the resolution by a factor of 4. Clearly the distribution becomes smoother at coarser resolution. Figure 6 shows the same data to the power of two. From this figure it can be seen that the mean of these values, and therefore the second-order moment (i.e. σ^2), decreases with increasing scale.

Figure 7 shows this dependence. The straight-line fit on the log-log plot gives the value for the scaling exponent, which in this case is $K(2) = 0.32$. Figure 8 shows a plot of the scaling exponent $K(q)$ as a function of the order of the moment, q.

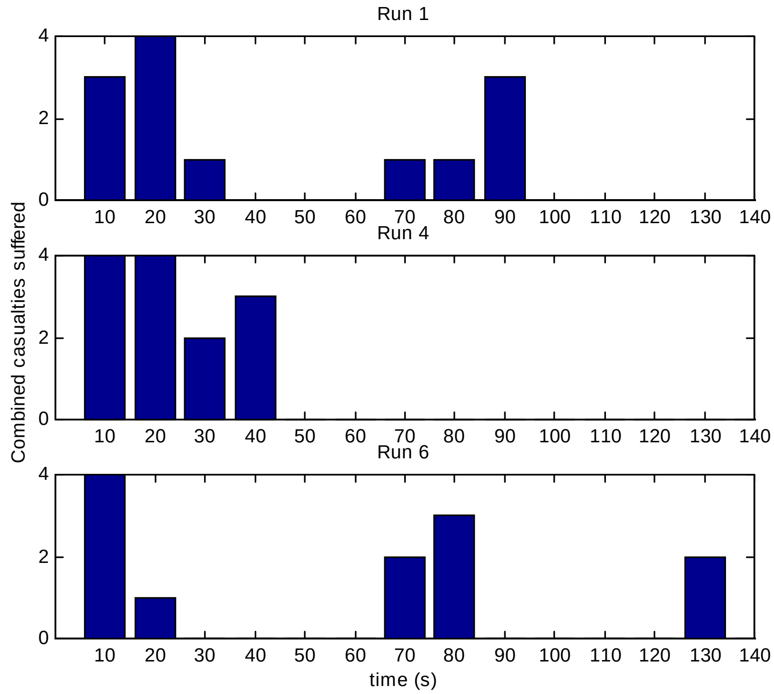

Figure 3: Blue casualties shown in bins of 10 second duration.

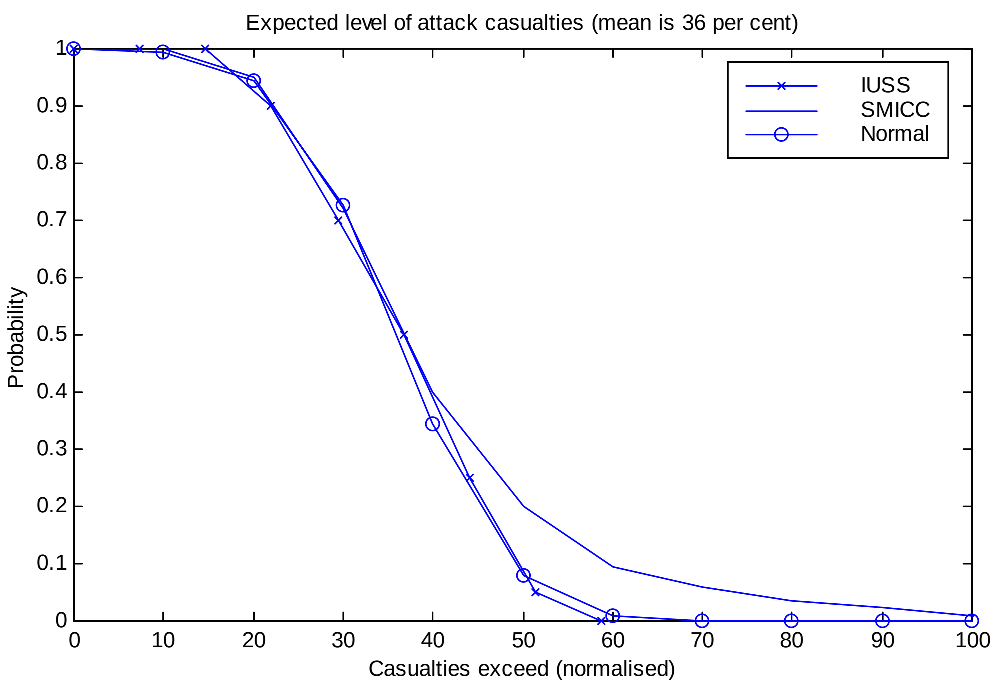

Figure 4: Exceedence distributions for IUSS model compared with SMICC (lognormal) and normal distributions.

Thus this simulated data shows evidence of scaling. However, the scaling range must be expected to be truncated by the limitations of the method for simulating the data.

For example, since the time series is made up of subsets of data, the temporal scaling cannot be expected to hold for time scales larger then the duration of a typical data subset. The limited resources (i.e. number of participants) available in each subset also limit the maximum likely casualty rate.

It must be noted that joining data subsets up like this is quite artificial. A better method might be to place each data subset at random positions within some large stretch of time, allowing some overlapping. As with the previous method, this simulates smaller sections of some larger unit becoming engaged over some extended period of time. However, constructing a large data set from smaller subsets using this method rather than the one examined in detail makes no difference to the scaling argument, but rather only affects the value of the parameters. In fact, this joining of the data set was only necessary to demonstrate how long scaling ranges occur.

The subsets scale by themselves, but only for a limited range in keeping with the brief nature of the encounter, so that the scaling is not an artefact of the construction of the data set.

3.2 A purely statistical approach

Having discussed the implications of using a discontinuous function such as a fractal to describe attrition, and seen that even for a largely deterministic model like IUSS there is some justification for using such a function, this section briefly details how such attrition data may be generated.

One method for producing data of this kind is to use a fractal cascade model (see Davis *et al*, 1994). This type of model produces simulated data by doing the opposite of figure 5. Instead of aggregating the data, the model starts with a uniform casualty rate R for the entire period, then splits the period in two and assigns the values W1R and W2R to each new interval respectively, where W_1 is a random weighting with some (empirically determined) probability distribution, and $W_1 + W_2 = 2$. This process is repeated enough times to produce a desired number of data points.

Figure 9 shows a simulated casualty time series generated using this cascade model with a normally distributed weighting factor. This distribution is illustrative of how the casualty rate distribution may be expected to appear if attrition has a multiscaling, fat-tailed distribution. Note that by reducing the resolution of such a time series by an arbitrary amount, a set of random variables representing casualty rates from a unit of an arbitrary size can be obtained in principle, which has the appropriate fat-tailed, intermittent and multiscaling probability distribution.

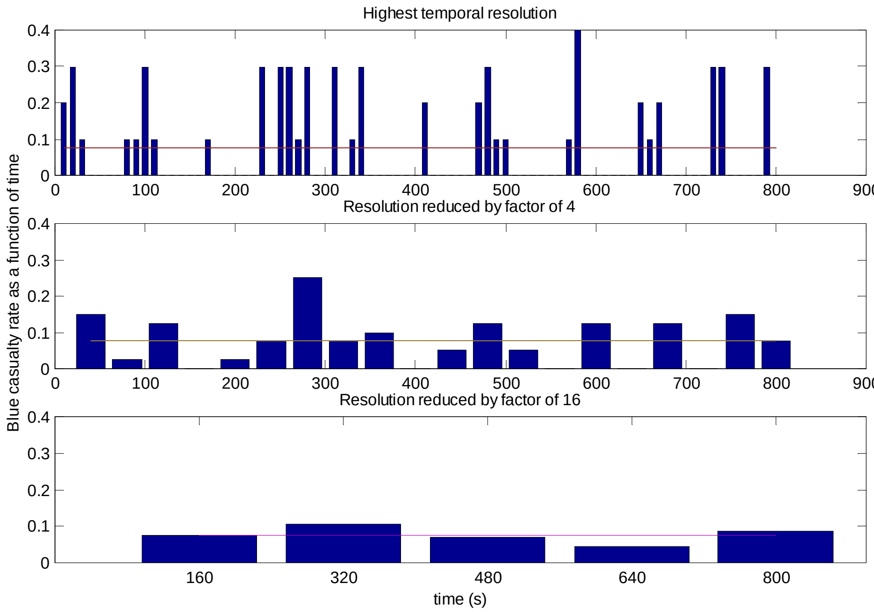

Figure 5: Casualty rate with decreasing resolution. The mean casualty rate, marked by the line across the plot, is 0.76 casualties per second.

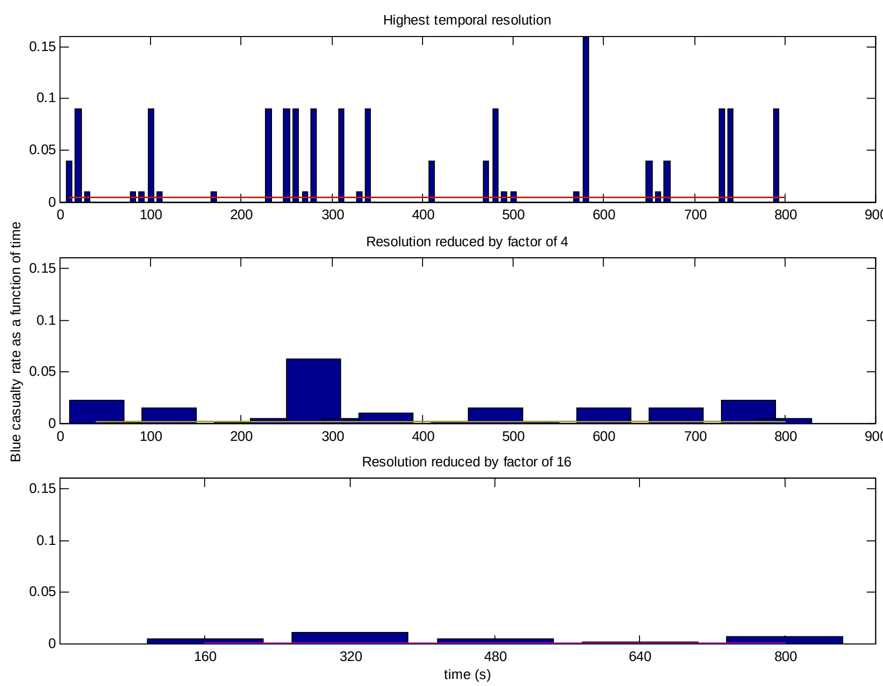

Figure 6: The casualty rate to the power of two. Note that the mean value (which is the second-order moment of the data), unlike figure 5, is not constant with scale.

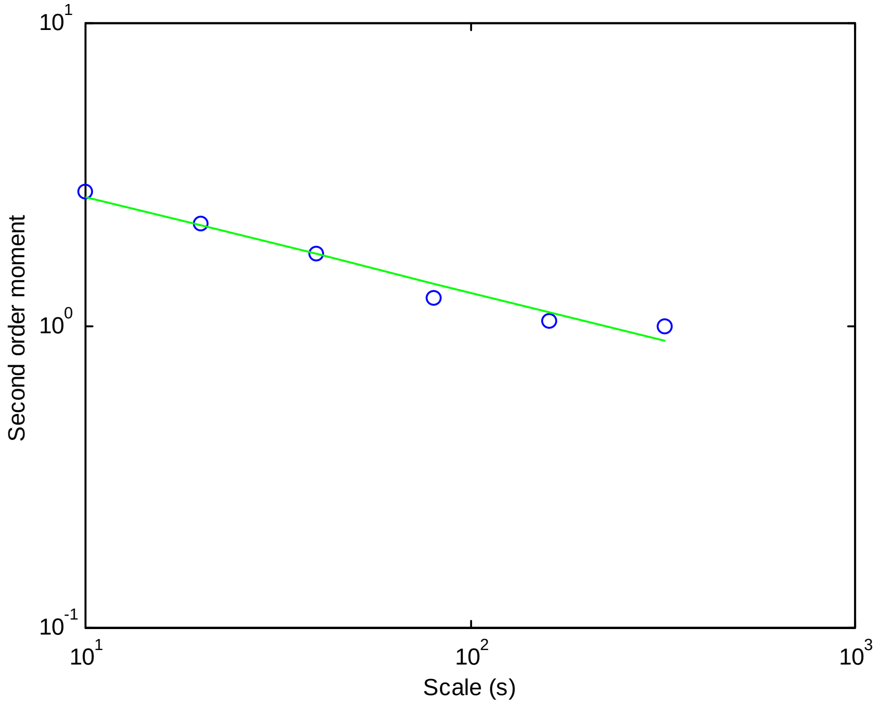

Figure 7: Scaling of the second-order moment of the casualty rate data. The slope of the plot, and hence the value for the scaling exponent is 0.32.

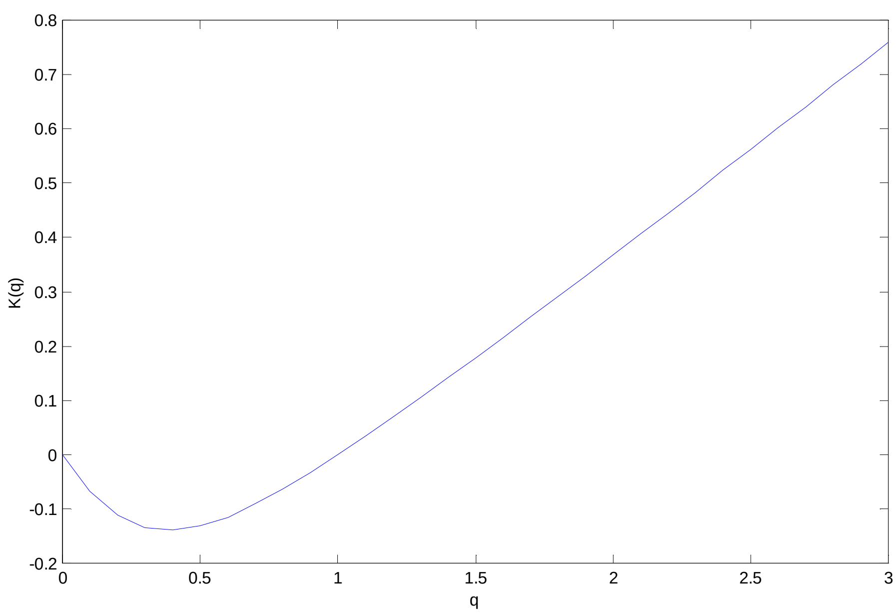

Figure 8: The value of the scaling exponent as a function of the order of the statistical moment.

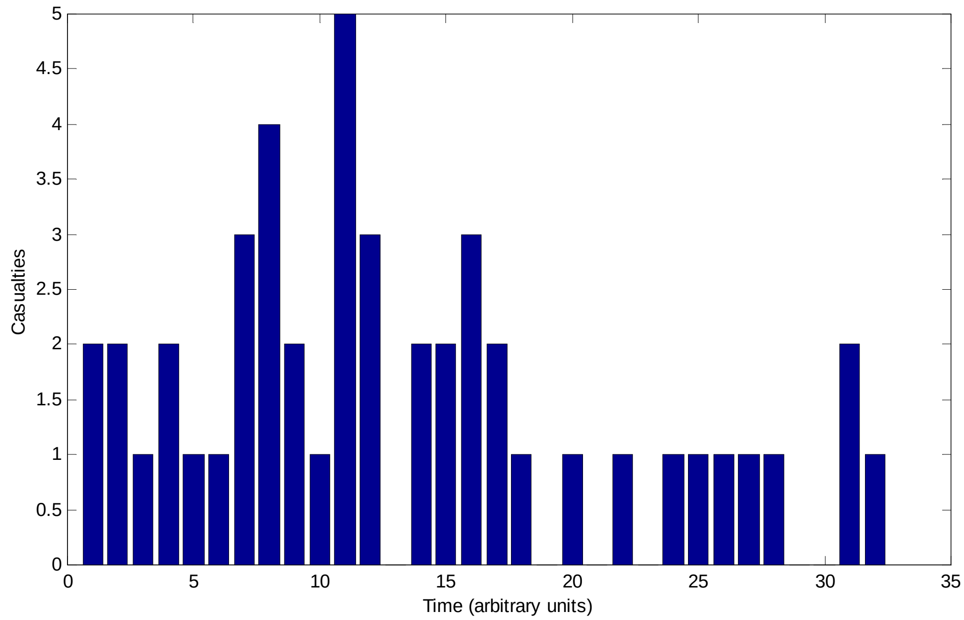

Figure 9: A sample simulated casualty distribution assuming that the attrition function is a multifractal.

4. Conclusions

This investigation examined the possibility that combat statistics are scaling, and hence have fractal indices. The main two consequences of this kind of statistical behaviour is that the combat should be more variable for smaller-scale forces, and that casualties should come in bursts. Both of these consequences seem reasonable. Though the fractional kill probabilities typically used in combat models may be expected to lead to a binomial distribution, geometry of the battlefield and tactics increases the clumpiness of casualties as fire is concentrated sequentially at different points. The variance of such clumpy data decreases as the data is increasingly aggregated, which is a property exhibited by fractals.

The historical data discussed and the simulated data presented suggest that the statistical properties of combat may be described in terms of fractal dimensions. Certainly the data from the IUSS simulation were quite discontinuous, and likely to lead to the existence of a fractal dimension and other scaling properties for the data.

Essentially, the existence of such scaling means that combat has structure on many scales, so that small-scale combat resembles large-scale combat. At each scale there is a battle and a campaign equivalent. At low levels, terminology tends to engagement and battle. At the lowest level, there are duels and series of duels. Attrition is during duels followed by manoeuvre in space and time. Tactics will try to concentrate duels for all individuals.

The existence of such a self-similar hierarchy provides phenomenological justification for modelling combat in terms of aggregated units, a common practice for war games. If combat outcomes for large units may be treated as an aggregation of the outcomes of combat between sub-units occurring on shorter characteristic time scales, the statistics of the outcomes are likely to be scale dependent. CDA's statistically based combat model SMICC does not consider this possibility, i.e., the variance remains constant as a proportion of the size of the unit. However, if the statistical moments of the data scale, it can be expected that the variance relative to the size of the force will increase as the size of the force decreases.

Note that the dependence of the statistical variance on the size of the force results from using an attrition rate which is not continuous. Thus this dependence may exist even if all a simulation does is line up two opposing forces and allow them to "shoot it out". The interesting question is then, what happens to the statistics if the opposing forces are allowed to position themselves in a dynamically evolving and non-linear way?

It is hypothesised here that the non-linearities inherent in real combat mean that the statistics should show behaviour typical of multiscaling fractal (multifractal) fields, such as fat-tailed probability distributions. This has significant implications for the risk associated with warfighting. Unfortunately, this hypothesis it difficult to prove without detailed data of real conflicts which is unlikely to become available. However, with the growing acceptance of automaton models as a method for simulating combat, which are designed to incorporate behaviour which adapts to the situation, it may be that a clearer link between fractal statistics and the non-linear dynamics of combat can be established.

Comparison of the statistical distribution of the IUSS model data examined here and the lognormal distribution used in CDA's SMICC suggests that detailed combat models do not necessarily reproduce the historically observed statistics of combat. The difference might be explained by the fact that IUSS fails to take into account the most unpredictable element of combat (the behaviour of the participants.

The results presented should not be taken as an absolute statement that combat between large forces is less unpredictable or complex than small forces. What it says is that the variance of the statistics gets smaller as the size of the units gets bigger for some range of unit sizes. For example, it might be argued that combat between two infantry sections is a small-scale version of what happens in combat between two infantry platoons, which is a small-scale version of what happens between two infantry companies, so scaling arguments may apply. But combat between two infantry battalions may not resemble two companies, because, hypothetically, battalion-level units involved in the conflict may relate to each other in a fundamentally different way to smaller units (e.g. the commander is less inclined to put the whole battalion at risk than a single company, particularly if the entire force only includes a few battalions). However, in a conflict with very large opposing forces, battalions might be treated similarly to companies in a small force.

If small-scale combat has a much higher degree of variance, as suggested here, explicit models of combat on this scale may often be of limited use for analytical purposes. At the very least, it must be expected that such simulations must be run hundreds of times in order to gain knowledge of the distribution of outcomes. On the other hand, large-scale combat outcomes are much less variable, suggesting that simulations of combat between large forces may be usefully predictive.

In this case, it might be just as useful for analytical purposes to use simple combat models which describe the evolution of the combat in terms of generalised events, provided these models can reproduce the appropriate statistics. Such simple models may be easily run a large number of times, thus providing a more complete picture of the statistical variations likely for small-scale scenarios.

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